Math 332 • Midterm Exam • March 9, 2016 • Victor Matveev

1) **(24pts)** Find **all distinct** values of *z*, in Cartesian or polar form. For parts (a) and (b), show the locations of these points in the complex plane. In (b), start from the definition of tan(z) in terms of exponential functions ($\tan z = \sin z / \cos z = ...$)

(a) $z=(1-i)^{4/3}$ (b) $\tan z = 2i$ (c) $z=(-i)^{1-i}$

2) **(32pts)** Calculate each integral over the given circle, or explain *clearly* why the integral equals zero; make sure to indicate the locations of singularities of each integrand:

a) $\oint\limits_{|z|=5} \frac{e^z\,dz}{\left(e^z-1\right)^9}$ b) $\oint\limits_{|z|=1} \frac{dz}{\cos z+1}$ c) $\oint\limits_{|z|=2} \frac{\sin\left(z^3\right)dz}{z^2+1}$ d) $\oint\limits_{|z|=4} \frac{dz}{\sqrt{z}}$

3) **(14pts)** Differentiate this function: $f(z)=(\cos z)^{\operatorname{Log} z}$

4) **(14pts)** Is the function $f(z)=\frac{(\bar{z})^2}{z}$ differentiable anywhere? Is it analytic anywhere? Is this function continuous in the entire plane? Use one of the following forms of Cauchy-Riemann equations in polar coordinates to analyze analyticity / differentiability:

$$\frac{df}{dz}=e^{-i\theta}\frac{\partial f}{\partial r}=-i\frac{e^{-i\theta}}{r}\frac{\partial f}{\partial\theta} \Rightarrow \text{or, written in component form} \Rightarrow \begin{cases} u_r = \dfrac{v_\theta}{r} \\ v_r = -\dfrac{u_\theta}{r} \end{cases}$$

============= **Pick 1 problem out of the last 2 (i.e. drop one problem)** =========

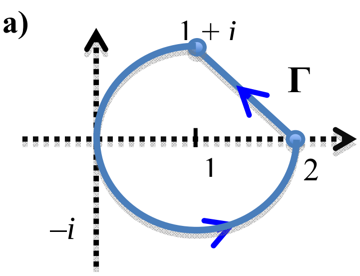
5) **(16pts)** Sketch the region π/2 ≤ Re *z* ≤ π, 1 ≤ Im *z* ≤ 2, and sketch its image under the transformation *w* = exp($i\bar{z}$). It may help to decompose this map into three elementary steps.

6) **(16pts)** Calculate the following integrals, using an appropriate method in each case, or explain why the integral is zero:

a) $\oint\limits_\Gamma \operatorname{Im}(z)\,dz$, where Γ is shown in the top figure

b) $\int\limits_\gamma \frac{z\,dz}{\left(z^2-1\right)^2}$, where γ is shown in the bottom figure

a)



b)